# Modeling of experimentally observed driven dust vortex characteristics in laboratory plasma

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# Outline

1. Introduction and Motivations
2. Modeling and fluid simulation of dust vortex characteristics
   - 2D hydrodynamic model of bounded dust flow in streaming plasma
   - Analytical solution of bounded dust vortex flow in $Re \leq 1$ regime
   - Bounded dust vortex characteristics in non-linear regime, $Re > 1$
3. Studies of dust vortex characteristics in magnetized dusty plasma
   - Dust vortex characteristics in weakly magnetized plasma
4. Summary and future work
5. Other relevant informations

# Introduction to complex/dusty plasma

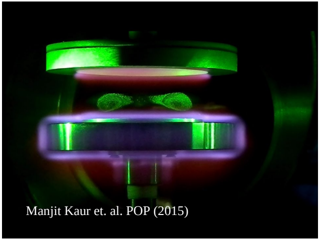
- Normal plasma + additional small sized particles.
- Particles get highly charged and shows collective behaviors.
- Typical glow discharge argon-dusty plasma parameters are,

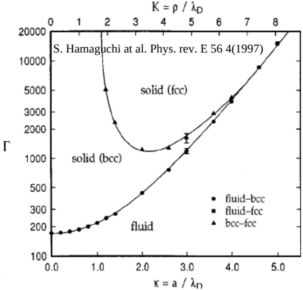
$Z_i n_i = n_e + Z_d n_d$, $Z_d \approx 10^4 e$, $m_d \approx 10^{-14}\ kg$, $m_i \approx 10^{-26}\ kg$, $n_d \simeq 10^3 \mathrm{cm}^{-3}$, $n_i \simeq 10^8 \mathrm{cm}^{-3}$, $n_n \simeq 10^{13} \mathrm{cm}^{-3}$ ($\approx$ 10 *pascal*), $T_i \simeq 1\ eV$, $T_e \simeq 3\ eV$, $c_{ds} \approx 12.65\ cm/sec$.

- Dust cloud can exist in various state of matters ($\Gamma$, $k$).

$$\Gamma (= PE/KE) = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{a\, k_B T} \exp(-\kappa)$$

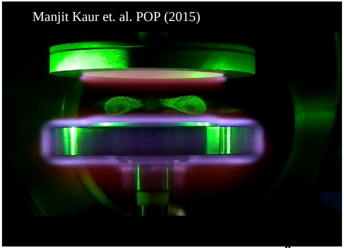
$$\kappa = \frac{a}{\lambda_D}; \quad a = (3/4\pi n)^{1/3},\ and\ \lambda_{Dj} = (K_b T_j / n_j e^2)$$

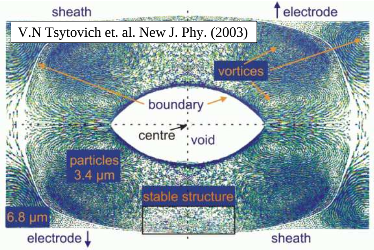


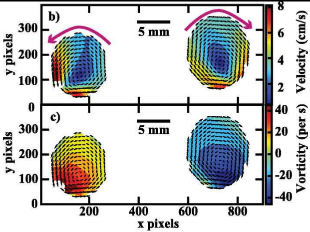



# Motivations;- vortex structures in dusty plasma

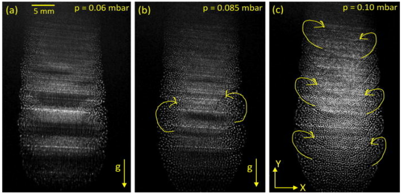
- Vortices are observed as a signature of various driven-dissipative systems.



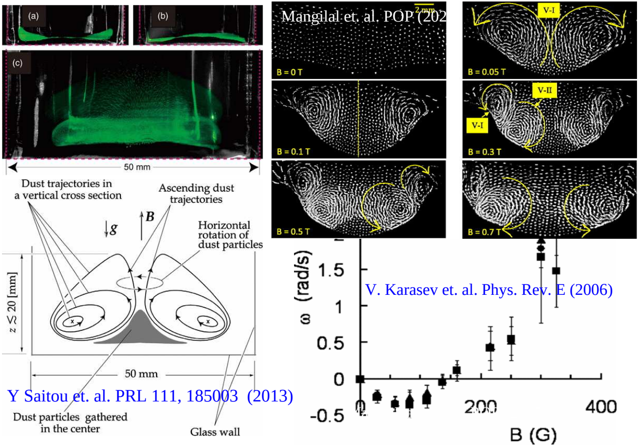

Mangilal et. al. POP (2018)




- The underlying physics requires systematic theoretical interpretations.

# Motivations;- vortices in magnetized dusty plasma

- Strange vortex structure are observed in magnetized dusty plasma.




- The role of $\mathbf{E} \times \mathbf{B}$, $\nabla \mathbf{B} \times \mathbf{B}$, $\nabla n_j \times \mathbf{B}$, and polarization drift in the vortex characteristics are to be interpreted systematically.

# “Multi-fluid model of complex plasma”

$$\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \mathbf{u}_j) = \eta_j n_j,$$

$$n_j m_j \frac{d\mathbf{u}_j}{dt} = \eta_j n_j m_j \mathbf{u}_j - \nabla p_j + q_j n_j (\mathbf{E} + \mathbf{u_j} \times \mathbf{B}) + \sum_k m_j n_j \nu_{jk} (\mathbf{u_j} - \mathbf{u_k}),$$

$$\frac{1}{\gamma_j - 1} \left( \frac{\partial p_j}{\partial t} + \mathbf{u}_j \cdot \nabla p_j \right) = -p_j \nabla \cdot \mathbf{u}_j - \nabla \cdot \mathbf{q}_j + S_j + \sum_k 2 n_j m_j \frac{m_j}{m_k} \nu_{jk} (T_j - T_k),$$

$$\nabla \cdot \mathbf{E} = \frac{\rho_q}{\epsilon_0},$$

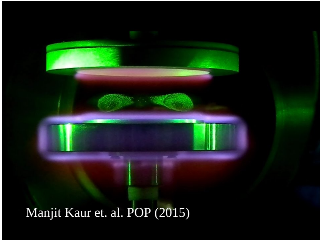
$$\nabla \cdot \mathbf{B} = 0,$$

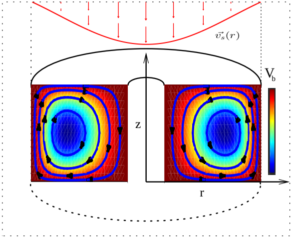
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t},$$

$$and \;\; \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}.$$

- These set of equations can be simplified under various approximations.
- Highly mobile electrons and ions are thermalized before the dust distribution maintains a steady flow.

# *"2D Hydrodynamic model of bounded dust flow in a plasma"*




$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\nabla\phi_b - \frac{\nabla P}{\rho} + \mu\nabla^2\mathbf{u} + f_s + f_B, \tag{2}$$

$$f_s = -\xi(\mathbf{u} - \mathbf{v_i}) - \nu(\mathbf{u} - \mathbf{w_n}), \quad and \quad f_B = \frac{q_d}{m_d}\mathbf{E} + \frac{1}{\rho}(\mathbf{J_d} \times \mathbf{B}). \tag{3}$$

- Using $\mathbf{u} = \nabla \times \psi\hat{\phi}$, $\omega\hat{\phi} = \nabla \times \mathbf{u}$, and $\mathbf{w_n} \approx 0$, the equation in an axisymmetric cross-section (r,z) of the toroidal setup is follows,

$$\nabla^2\psi = -\omega, \qquad \frac{\partial \omega}{\partial t} + (\mathbf{u} \cdot \nabla)\omega = \mu\nabla^2\omega - (\xi + \nu)\omega + \xi\omega_s + \beta\omega_B.$$

- Dust dynamic depends on $\mu$, $\xi$, $\nu$, $\beta$, $\omega_s$, $\omega_B$, and nature of boundaries.

# Calculation of system parameters $\xi$, $\nu$, $\mu$ and $\omega_s$;

$$\nabla^2\psi = -\omega, \qquad \frac{\partial\omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \cancel{\beta\omega_B}.$$

• Using conservation of momentum between colliding particles,

$$m_d n_d(u_d - v_j)\nu_{dj} = -m_j n_j(v_j - u_d)\nu_{jd}$$

$$\nu_{dj} = \frac{m_j n_j}{m_d n_d}\nu_{jd}, \quad \nu_{jd} = n_d v_j \sigma_{jd}.$$

• For kinematic viscosity, $R_e (= L_r u_d/\mu) \simeq 1$

• **M. S. Barnes** *et al.*, Phys. Rev. Lett. 68, 313 (1992), **S. A. Khrapak** *et al.*, Phys. Rev. E 66, 046414 (2002).

Further,

$$\sigma_{nd} = \pi r_d^2, \quad \sigma_{id} = 4\pi b_{\pi/2}^2 \ln\Lambda, \quad b_{\pi/2} = \left(\frac{1}{4\pi\epsilon}\right)\frac{Ze^2}{K_B T_i},$$

• $\xi\omega_s = \nabla\times(\xi\mathbf{v_i})$, $\quad \xi = 4\pi\frac{m_i n_i v_i}{m_d}\left(\frac{Ze^2}{4\pi\epsilon K_B T_i}\right)^2 \ln\Lambda.$

• Therefore, $\omega_s$ is combination of non-zero shear flow fields $(\nabla\times\mathbf{v_{i(n)}})$, $(\nabla Q_d \times \mathbf{E})$, $(\nabla v_{i(n)} \times \nabla n_{i(n)})$, and others .

# Analytical solution in the linear limit ($Re \leq 1$) ;-

$$\nabla^2\psi = -\omega, \qquad \frac{\partial\omega}{\partial t} + \cancel{(\mathbf{u}\cdot\nabla)}\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \cancel{\beta\omega_B}.$$

- In the linear regime($Re < 1$), the set of equations become,

$$\nabla^2\psi = -\omega, \quad \nabla^2\omega - K_1\omega + K_2\omega_{\mathbf{s}} = 0, \;\; K_1 = (\xi+\nu)/\mu, \;\; K_2 = \xi/\nu. \tag{4}$$

$$\frac{\partial^4\psi}{\partial r^4} + \frac{2}{r}\frac{\partial^3\psi}{\partial r^3} - \left[\left(\frac{3}{r^2} + K_1\right) - \frac{2\partial^2}{\partial z^2}\right]\frac{\partial^2\psi}{\partial r^2} + \left[\left(\frac{3}{r^3} - \frac{K_1}{r}\right) + \frac{2}{r}\frac{\partial^2}{\partial z^2}\right]\frac{\partial\psi}{\partial r}$$
$$- \left[\frac{3}{r^4} - \frac{K_1}{r^2} + \left(\frac{2}{r^2} + K_1\right)\frac{\partial^2}{\partial z^2} - \frac{\partial^4}{\partial z^4}\right]\psi - K_2\omega_{\mathbf{s}} = 0. \tag{5}$$

- Solved Numerically using MATLAB solvers such as **ode45, pdepe, bvp4c.**
- Solved Analytically using **Fourier series expansion, Eigenvalue method**.

$$\psi = \psi_r\psi_z;\; \psi_r = \sum_{m=1}^{\infty} a_m J_n\left(\alpha_m\frac{r}{L_r}\right), \; \psi_{sr} = \sum_{m=1}^{\infty} b_m J_n\left(\alpha_m\frac{r}{L_r}\right), \; \psi_z = \sum_{n=1}^{\infty} a_n\cos(k_n z)$$

★ Laishram, Sharma, and Kaw, *Phys. Rev. E* **91**, 063110 (2015).

# Eigenvalue formulation in cylindrical coordinate ;

- The above-combined equation is reduced to a simple Eigenvalue problem as given below.

$$\sum_{m=1}^{\infty} (\lambda_m a_m - K_2 b_m) J_n\left(\alpha_m \frac{r}{L_r}\right) = 0, \tag{6}$$

The set of equations for $M - modes$ can be rearranged in a more familiar form,

$$\begin{bmatrix} A_{11} & A_{12} & \dots & A_{1M} \\ A_{21} & A_{22} & \dots & A_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1} & A_{M2} & \dots & A_{MM} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_M \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_M \end{bmatrix} \tag{7}$$

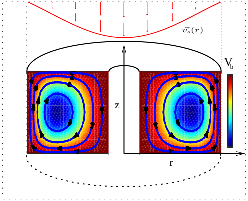
where, $A_{ij} = \lambda_j J_n(\alpha_j r_i / L_r)$, and $B_i = K_2 \sum_{j=1}^{M} b_j J_n(\alpha_j r_i / L_r)$.

- Using proper boundary conditions ($u_\perp = 0,\ u_\parallel = ?$), the above equations are solved for coefficients $a_j$. Then solve for $\psi_r$, and finally we get $\psi = \psi_r(r)\psi_z(z)$.

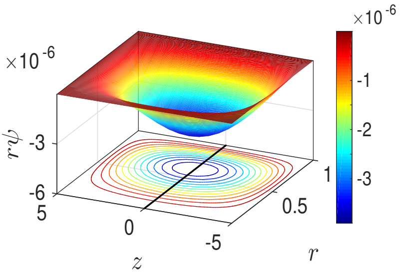
# Flow solutions and effect of boundary conditions;

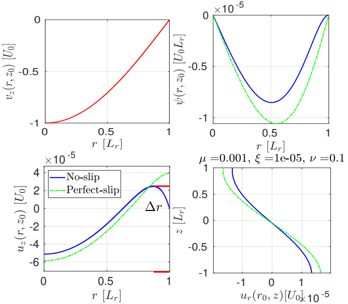
$$\nabla^2\psi = -\omega, \qquad \nabla^2\omega - K_1\omega + K_2\omega_s = 0. \qquad (Re < 1) \tag{8}$$

- Let the background sheared ions follow natural Bessel mode as follows,



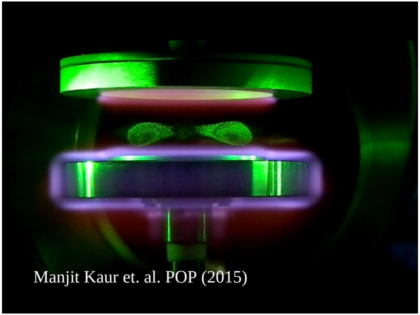

$$\mathbf{v_z(r,z)} \quad = \quad U_a + U_0 J_0\left(\alpha_m \frac{r}{L_r}\right).$$




- No-slip boundary $u_{||} = 0.0$ , introduces boundary layer formation.
- $u_d \leq 1.0\ cm/sec$, while the acoustic velocity($\sim U_0$) $\approx 12.65\ cm/sec$.

# "Nonlinear effects in the bounded dust vortex flow in plasma"



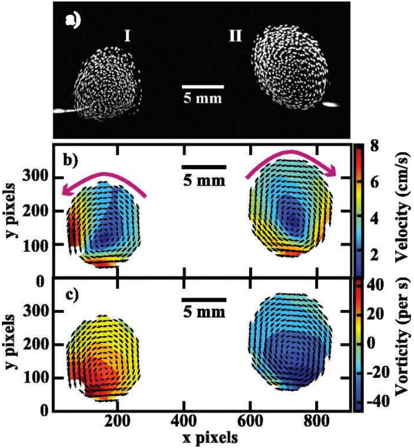




Figure: Dust vortex in Experimental lab(IPR), by M. Kaur et. al. Phys. of Plasma **22**, 033703 (2015).

$$\nabla^2\psi = -\omega, \qquad \frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \cancel{\beta\omega_B}.$$

★ Laishram, Sharma, Prabal, and Kaw, *Phys. Rev. E* **95**, 033204 (2017).

# Dust flow solution in non-linear regime $Re \geq 1$ ;-

• The above dust dynamical formulation is extended to higher Reynolds number nonlinear flow regimes ($Re \geq 1$) .

$$\nabla^2\psi + \omega = 0,$$

$$\nabla^2\omega - K_1\omega + K_2\omega_{\mathbf{s}} - \frac{1}{\mu}(\mathbf{u}\cdot\nabla)\omega = 0. \qquad (Re \geq 1)$$

• Using SOR-Iterations method, the above set of equations are solved using proper boundary conditions ($u_\perp = 0,\ u_\parallel = ?$).

$$\frac{\Delta\psi}{\Delta L^2} \approx R_\psi = \nabla^2\psi + \omega$$

$$\psi^{n+1} = \psi^n + \Delta L^2\nabla^2\psi^n + \Delta L^2\omega^n,$$

$$\textit{Similarly}, \quad \omega^{n+1} = \omega^n + \Delta L^2\nabla^2\omega^{n+1} - \Delta L K_1\omega^n + \ldots\ldots$$

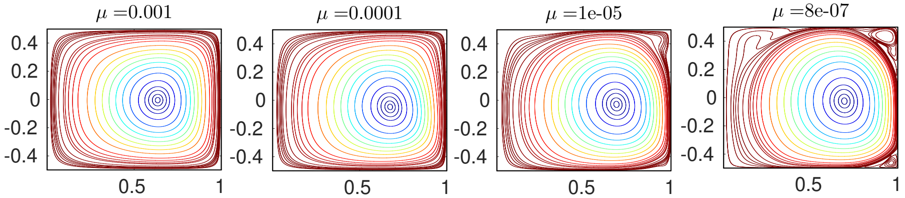
• This numerical formulation is benchmarked with well-known fluid flow problems and previous analytical solutions in the linear regime ( $Re < 1$).

★ Laishram and Zhu, *Physics of Plasma* **25**, 103701 (2018).

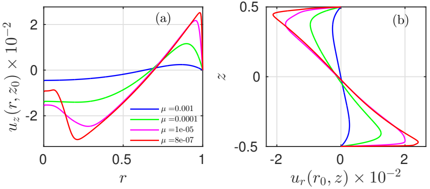
★ Laishram, Sharma, and Kaw, *AIP Conf. Proc.* **1925** (2018).

# Nonlinear characteristics in domain of $L_z/L_r = 1$ ;

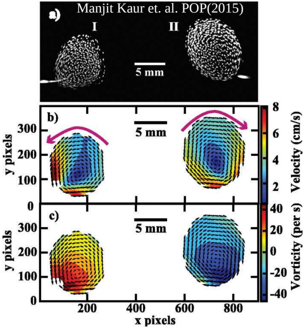
$$\nabla^2\psi = -\omega, \qquad \frac{\partial\omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \cancel{\beta\omega_B}.$$




- Flow structure turns into circular patterns in highly nonlinear regimes.



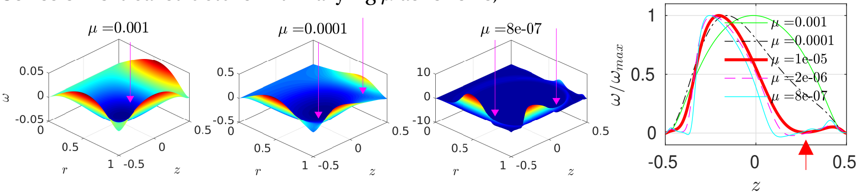

- Uniform vorticity core region surrounded by highly shear layers is the nonlinear characteristic of the flow.



# Nonlinear structural bifurcation and scaling laws ;

$$\nabla^2\psi = -\omega, \qquad \frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \cancel{\beta\omega_B}.$$

- Series of vortical structure with varying $\mu$ as follows,

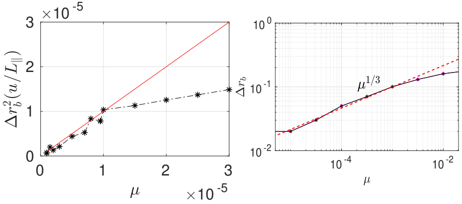



$\Rightarrow$ The critical parameter $\mu \sim \mu^*$ corresponds to degenerate singular point ($\omega_b = 0,\ \omega_b' = 0$) of the flow field at the boundaries that bifurcates into two isolated solutions through the $\mu^*$.

- Recovered scaling laws;

$$\mu \approx \Delta r_b^{1/3} \qquad (\mu \gg \mu^*)$$

$$\Rightarrow \mu \approx \Delta r_b^2\left(\frac{u}{L_{\|}}\right), \qquad (\mu \le \mu^*)$$




- These scaling laws can estimate the kinematic viscosity of the driven dust flow.

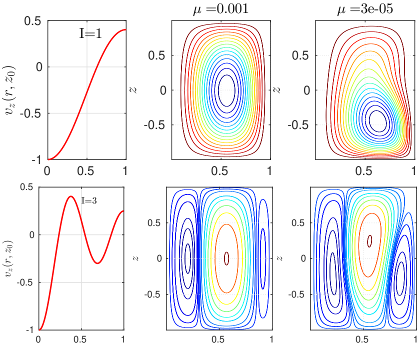
★ Laishram, Sharma, Prabal, and Kaw, *Phys. Rev. E* **95**, 033204 (2017).

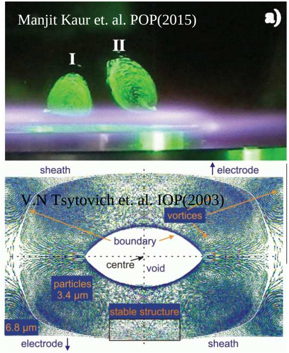
# Nonlinear characteristics in domain of $L_z/L_r = 2$ ;

$$\nabla^2\psi = -\omega, \qquad \frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \cancel{\beta\omega_B}.$$

- Let the background sheared ions follows higher natural Bessel modes.

$$\mathbf{v_z}(\mathbf{r},\mathbf{z}) = U_a + U_0 J_0\left(\alpha_I \frac{r}{R}\right), \qquad I = 1,\ 2,\ 3,\ 4....$$

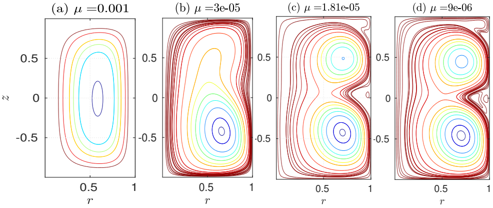


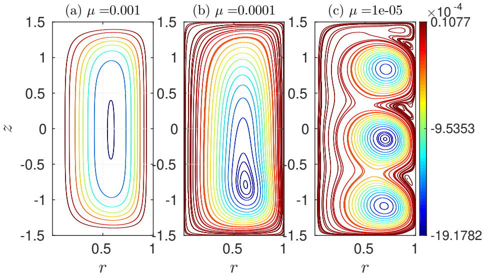

- Dominant scales are introduced by the driving fields and boundaries.

★ Laishram and Zhu, *Physics of Plasma* **25**, 103701 (2018).

# Conditions for steady state co-rotating vortices :




| $L_z : L_r$ | $\mu^*$ $[U_0 L]$ |
|---|---|
| 3 | $8 \times 10^{-5}$ |
| 2 | $2 \times 10^{-5}$ |
| 1 | $1 \times 10^{-5}$ |
| 0.5 | $2 \times 10^{-6}$ |



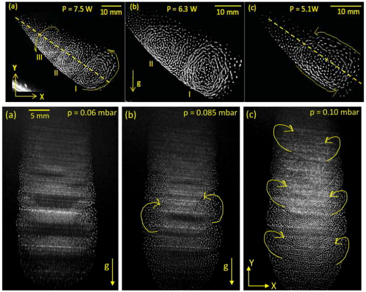
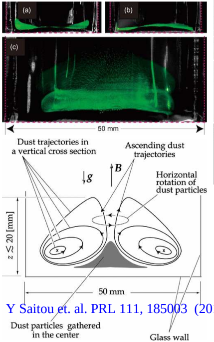




**Figure:** Steady state co-rotating vortices observed in dusty plasma experiments by Mangilal et al, Phys. Plasma **24**, 033703 (2017).

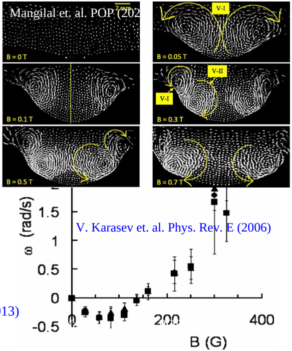
- Co-rotating vortices are the outcome of nonlinear structural bifurcation through a threshold parameter $\mu^*$.

★ Laishram, Sharma, and Zhu, *Phys. D; Applied Physics* **21**, 073703 (2019).

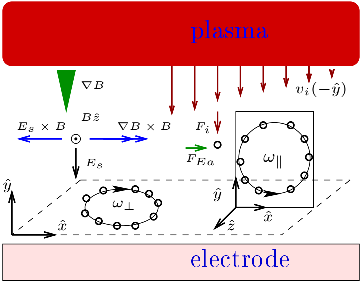
# *“Bounded dust vortex flow structure in magnetized plasma”*




Y Saitou et. al. PRL 111, 185003 (2013)




V. Karasev et. al. Phys. Rev. E (2006)




$$\nabla^2\psi = -\omega, \qquad \frac{\partial\omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \beta\omega_B.$$

Here, $\xi\omega_s = \nabla\times(\xi\mathbf{v_i})$, $\beta\omega_B = \nabla\times\left[\frac{q_d}{m_d}\mathbf{E} + \cancel{\frac{1}{\rho}(\mathbf{J_d}\times\mathbf{B})}\right]$, $\mathbf{E} = \mathbf{E}_s + \mathbf{E_a}$, and $\mathbf{E_a} = ?$

★ Laishram, https://arxiv.org/abs/2011.03237 (2020).

# Derivation of ambipolar field $\mathbf{E_a} = ?$

- Starting from the flow equation of electrons and ions across the $\mathbf{B}$, we have,

$$u_{j\perp} = \pm\mu_{j\perp}\mathbf{E}_\perp - D_{j\perp}\frac{\nabla n_j}{n_j} + \frac{\mathbf{v}_{jB} + \mathbf{v}_{jD} + \mathbf{v}_{j\nabla B}}{1 + (\nu_{jn}^2/\omega_{jc}^2)}. \tag{9}$$

- Using $n_i u_{i\perp} = n_e u_{e\perp}$ and $n_i = n_e + Z_d n_d$ in the regime $\omega_{ec} \geq \nu_{en}$ **to** $\nu_{in} \geq \omega_{ic}$, the expression for $E_a$ and $\beta\omega_B = \nabla \times \frac{q_d}{m_d}\mathbf{E_a}$ are derived as

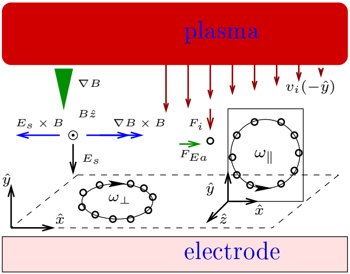
$$\mathbf{E_a} = \eta q_j(D_{j\perp}\nabla n_j) + \frac{\eta q_e}{(1 + \nu_{en}^2/\omega_{ec}^2)}\left[n_e\frac{(\mathbf{E}\times\mathbf{B})}{|\mathbf{B}|^2} + \frac{K_bT_e}{q_e}\frac{(\nabla n_e \times \mathbf{B})}{|\mathbf{B}|^2} + \frac{n_eK_bT_e}{q_e}\frac{\nabla\mathbf{B}\times\mathbf{B}}{|\mathbf{B}|^3}\right].$$

$$\begin{aligned}\beta\omega_B = \frac{\eta q_e q_d}{m_d(1 + \nu_{en}^2/\omega_{ec}^2)}\Bigg[ &- n_e\left(\frac{\mathbf{B}}{|\mathbf{B}|^2}(\nabla\cdot\mathbf{E}) - (\frac{\mathbf{B}}{|\mathbf{B}|^2}\cdot\nabla)\mathbf{E} + (\mathbf{E}\cdot\nabla)\frac{\mathbf{B}}{|\mathbf{B}|^2}\right)\\ &-\frac{K_bT_e}{q_e}\left(\frac{\mathbf{B}}{|\mathbf{B}|^2}(\nabla^2 n_e) - (\frac{\mathbf{B}}{|\mathbf{B}|^2}\cdot\nabla)\nabla n_e + (\nabla n_e\cdot\nabla)\frac{\mathbf{B}}{|\mathbf{B}|^2}\right)\\ &-\frac{n_eK_bT_e}{q_e}\left(\frac{\mathbf{B}}{|\mathbf{B}|^3}(\nabla^2\mathbf{B}) - (\frac{\mathbf{B}}{|\mathbf{B}|^3}\cdot\nabla)\nabla\mathbf{B} + (\nabla\mathbf{B}\cdot\nabla)\frac{\mathbf{B}}{|\mathbf{B}|^3}\right)\Bigg].\end{aligned}$$

- Further, we can simplify for $\beta\omega_{B\|}$ and $\beta\omega_{B\perp}$ in the driven system.

# Vorticity sources along with B i.e., $\beta\omega_{B\|}$

- In the cross-section **xy**, we have $\mathbf{E_s}(-\hat{y})$, $\nabla\mathbf{B}\hat{y}$, $\mathbf{B}\hat{z} = B_0 sin(yy)$, $yy = k_y\frac{y-y_1}{L_y-y_1}$, and $\mathbf{v}_i(-\hat{y}) = U_0\cos(xx)$, $xx = k_x\frac{x-x_1}{L_x-x_1}$.




- The vorticity sources are found to be,

$$\omega_s = \nabla \times \mathbf{v}_i = -U_0\frac{k_x}{L_x - x_1}\sin(xx),$$

And,

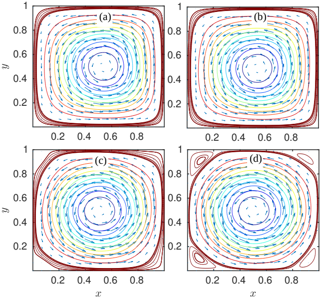
$$\beta\omega_{B\|} = -\eta\frac{n_e q_e q_d}{m_d(1+\nu_{en}^2/\omega_{ec}^2)}\left[\left(\frac{-E_s}{B_0}\right)\left(\frac{k_y}{L_y - y_1}\right)\frac{cos(yy)}{sin^2(yy)} + \left(\frac{K_b T_e}{q_e B_0}\right)\left(\frac{k_y}{L_y-y_1}\right)^2\left(\frac{1}{sin(yy)} - \frac{2cos^2(yy)}{sin^3(yy)}\right)\right].$$

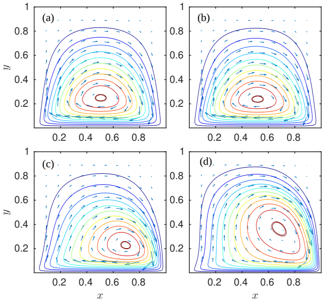
- In $\beta\omega_{B\|}$, the last term (due to $\nabla\mathbf{B}$) is six orders larger than the second term (due to $\nabla^2\mathbf{B}$) and three orders larger than the first term (due to $\mathbf{E_s} \times \mathbf{B}$).

# Driven dust vortex characteristics;

$$\nabla^2\psi = -\omega, \qquad \frac{\partial\omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \beta\omega_B.$$

- The equations are solved in the rectangular domain $0 \leq x/L_x \leq 1$, $0 \leq y/L_y \leq 1$, and $L_x/L_y = 1$ for a wide range of $\mu$ (or $\nu$, $\xi$, and $\beta$).



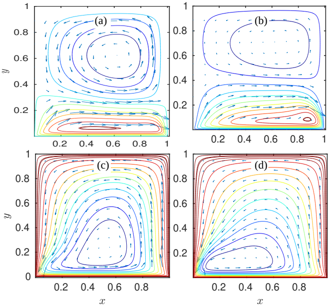



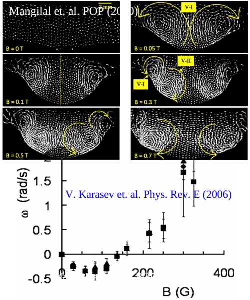

- $\omega_s$ generates a volumetrically driven **anti-clockwise circular** structure.
- $\omega_{B||}$ gives rise to **clockwise** ***D*-shaped elliptical** structure which turns into a meridional structure with varying parameters.

# Dust vortex structure at high pressure and low **B**;

$$\nabla^2\psi = -\omega, \qquad \frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \beta\omega_B.$$

- When both the $\omega_s$ and $\omega_{B\|}$ are comparable, a counter-rotating vortex pairs associated with different sources can co-exist in the same domain.




- When the **B** is reversed, both the sources act together and generate a strong meridional structure. The interior static point, $u(x_0, y_0) = 0$ follows,

$$0 = -\nabla\phi_b + \frac{q_d}{m_d}\mathbf{E_a} - \frac{\nabla P}{\rho} + \mu\nabla^2\mathbf{u} + \xi\mathbf{v} + \nu\mathbf{w}.$$

- In all the above analyses, $u_d \leq 6.0\ cm/sec$ and $c_{ds} \approx\ 12.65\ cm/sec$.

# Summary and future work

• We have developed a 2D hydrodynamic model for characterizing vortices in driven-dissipative flow systems in cartesian and cylindrical setup.

$$\nabla^2\psi = -\omega, \qquad \frac{\partial \omega}{\partial t} + (\mathbf{u}\cdot\nabla)\omega = \mu\nabla^2\omega - (\xi+\nu)\omega + \xi\omega_s + \beta\omega_B.$$

• The dynamical model is extendable for studies of wide ranges of magnetized dusty plasma such as **weak to strong** and **axial to transverse** magnetized plasma.

• Adding compressibility, the model may describe the formation of plasma spiral vortex, void-vortex pair, and associated transient phenomena such as wave, instabilities, and turbulent flows as reported recently.

• The 2D $\psi - \omega$ formulation ($\nabla^2\psi = \omega$) is isomorphic to the 2D drift-Poisson equations ($\nabla^2\phi = 4\pi en,\ \mathbf{v} = -\frac{c}{\mathbf{B}}\nabla\phi\times\hat{z},\ \omega = \nabla\times\mathbf{v} = n\frac{4\pi ce}{\mathbf{B}}\hat{z}$), i.e., $\phi \leftrightarrow \psi$ and $n \leftrightarrow \omega$. Further, the incompressible flow field ($\nabla\cdot\mathbf{v} = 0$) is similar to $\mathbf{B}$ patterns ($\nabla\cdot\mathbf{B} = 0$), i.e., $\mathbf{v} \leftrightarrow \mathbf{B}_\theta$ and $\psi \leftrightarrow \psi_\theta$ .

# Important relevant informations ;-

- Working in **Generalized Hybrid Kinetic-MHD model for burning plasma.**

★ **M. Laishram**, Zhu, and Hou, https://arxiv.org/abs/1911.01741 (2020).

- Working experience in parallel **NIMROD** hybrid kinetic-fluid code.

- Collaborating in **Modeling of dust particles transport in EAST-tokamak** using DTOKS-U transport code and HERMES plasma code.

★ *Dust Particles Preceding Vertical Displacement Events in EAST*, Luke Simons, Sanjib Sarkar, Rui Ding, **M. Laishram**, and others**(ongoing)**.

- Several invited talks, oral presentations, research publications, and awards.

**"Asian Under-30 Young Scientist and Student Award-2018"**

- Member of APS, EPS, AIP, IOP, PSSI, and AAPPS-DPP.

- **For more detailed information**, please refer to my CV.

**Thank you !**